Social Identity, Electoral Institutions and the Number of Candidates

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British Journal of Political Science 2010
Supplementary Materials

1 Appendix: Proofs of Propositions

Proof of Proposition 1. Any group A (B) citizen who fails to enter when her group does not have a candidate violates the behavioral prescription against harming the group's electoral performance, incurring a sufficiently large cost that there is always an incentive to enter. So $(0, n)$ and $(n, 0)$ are not possible for any n. ■

Proof of Proposition 2. In any $(1, n)$ setting, candidate A wins regardless of her policy, which is unconstrained by B candidates' choices. Given this and since $n > 1$, B candidates will only have an incentive to stay in if they win a share of group leadership. As such, in any $(1, n)$ equilibrium, there must be an n-way tie among group B candidates. An A citizen at the ideal point of the A "incumbent" could enter and tie the election without affecting the winning policy (since $n \geq 2$ and $A > B$, $A/2 > B/n$) earning utility $\gamma/2 - c > 0$, so there is an incentive to enter and $(1, n)$ is not possible for $n > 1$. ■

Proof of Proposition 3. First suppose $A \in (\frac{1}{2}, \frac{2}{3})$. For $(1, 1)$ equilibrium existence, the (i) A and (ii) B candidates must wish to stay in, and no other (iii) A or (iv) B candidates must wish to enter. (i) and (ii): Along with the fact that there can be no identity reason for a solo group candidate to exit, $\gamma > c$ ($g(B) > c$) implies the A (B) candidate won't drop out. (iii) There is no identity incentive for A entry. Suppose the A candidate's policy is at the median A voter. Then a potential A entrant must always lose, either to incumbent A (paying entry cost with no policy or winning benefit), or to the B candidate (suffering lexicographic identity losses), since $B > \frac{A}{2}$. So an A incumbent at the A voter median can deter entry by A citizens. (iv) There is no identity incentive for B entry. Suppose the B candidate is at the median B voter. Then a potential B entrant can do no more than tie for group B support, without affecting policy, which will not be worth the cost of entry so long as $c > \frac{g(B)}{2}$. Thus (i)-(iv) can be simultaneously satisfied and so $(1, 1)$ is possible. Now suppose $A \in (\frac{2}{3}, 1)$. Now an A citizen who shared the incumbent A's policy would be able to enter and win vote share $\frac{A}{2}$, tying for first since now $\frac{A}{2} > B$. Since $\frac{\gamma}{2} > c$ there will be an incentive to enter so $(1,1)$ is not possible. ■

Proof of Proposition 4. In a $(2,1)$ setting, if the two A candidates are not tied, then the trailing candidate, who pays entry costs but does not influence policy or receive winning benefits, will drop out because there will never be identity costs for doing so. So if $(2,1)$ is possible, it must involve a tie between the two A candidates.

Suppose that $A \in (\frac{1}{2}, \frac{2}{3})$. Each A candidate wins vote share $\frac{A}{2} < B$, so B wins the race and either A candidate would wish to exit for identity reasons to ensure victory for the other A candidate. So $(2,1)$ is not possible. Now suppose $A \in (\frac{2}{3}, 1)$. For $(2,1)$ equilibrium existence, the (i) A and (ii) B candidates must wish to stay in, and no other (iii) A or (iv) B candidates must wish to enter. (i) and (ii): Because there are clearly no identity reasons for exit, as exit would improve neither group vote share nor the probability of group victory, and because the A candidates tie with vote share $\frac{A}{2} > B$, $\frac{\gamma}{2} > c$ ($g(B) > c$) implies the A (B) candidates wouldn't drop out. (iii) There are clearly no identity incentives for A entry. Because they must tie, the two A candidates must be symmetrically spaced around the median A voter; they cannot be at the same position, since an arbitrarily close potential A entrant could get at least arbitrarily close to half of the A vote (because the distribution of ideal points is continuous), and win since $\frac{A}{2}$ defeats the B candidate. In Proposition 2 of Osborne and Slivinski, two incumbents spaced around a median voter can deter entry by a citizen who cares about winning and policy in the same way that ours do; the competition by group A candidates for group A voters takes on the same form here, except that additional constraints are imposed (for example, winning group A does not imply victory, as one must also defeat group B candidates in order to win). As such their deterrence result implies that all potential A entrants can be deterred here as well. (iv) There are clearly no identity incentives for B entry. Suppose the B candidate is positioned at the median B voter. Then a potential B entrant can do no more than tie for group B support, without affecting policy or identity, which will not be worth the cost of entry so long as $c > \frac{g(B)}{2}$. Note that (i)-(iv) can be simultaneously satisfied; so $(2,1)$ is possible. ■

Proof of Proposition 5. In a $(2,2)$ setting, if the two A candidates are not tied, then the trailing candidate, who pays entry costs but does not win or influence policy, would pay no identity costs for exiting, and will wish to. So any equilibrium must involve a tie between the two A candidates. Further, because either A candidate could ensure the victory of the other by dropping out, A identity concerns imply that all B candidates lose for sure in equilibrium. Given this and since there are multiple B candidates, B candidates can be motivated only by group leadership payoffs, as their policies have no effect on the A candidates. As any B candidate not tied for the lead would then wish to drop out, both group B candidates must win vote share $\frac{B}{2}$.

For $(2,2)$ equilibrium existence, the (i) A and (ii) B candidates must wish to stay in, and no other (iii) A or (iv) B candidates must wish to enter. (iv) There can be no identity motivation for B entry as all A candidates beat all B candidates. Since B entrants cannot affect policy outcomes, entry incentives are limited to B group leadership. The only way for tied B candidates to deter entry is with candidates symmetrically spaced about the median. (ii) There will be identity incentives to drop out if this increases the probability of a B candidate victory. For $(2,2)$ this is true if $B > \frac{A}{2}$, that is $A < \frac{2}{3}$. If there is no such identity incentive then candidates will wish to stay in so long as $\frac{g(B)}{2} > c$. (i) The A candidates tie for the win for sure, so there are no identity reasons for exit. Further, $\frac{\gamma}{2} > c$ ensures that neither A candidate will wish to exit. (iii) By the same logic as in part (iii) of the proof of Proposition 4, it is possible to deter entry by further A citizens. Note that (i)-(iv) can be simultaneously satisfied, and equilibria are therefore possible, when $A > \frac{2}{3}$ for $(2, 2)$. ■

Proof of Proposition 6. $(3, 1)$ configurations potentially involve: (1) An A candidate wins outright; (2) Two or more A candidates tie for the win; (3) An A candidate and the B candidate tie for the win; (4) Two or more A candidates and the B candidate tie for the win; and (5) the B candidate wins outright. But in (4), at least one of the A candidates would wish to drop out for identity reasons, to increase the probability of an A candidate winning. And in (1) or (3), there are two A candidates who lose outright, and who therefore do not share the winning A's policy. If the losing A's share the same policy, either of these will wish to drop out, whereas if they do not share the same policy, at least one of them (and possibly both) must not be the centrist candidate and will wish to drop out because they experience no winning, policy, or identity benefit from entry. If (5), the B candidate wins outright, and the losing A candidates do not even influence policy, so to stay in they must win some group leadership benefit and not wish to exit for identity reasons. Because of the former all three A candidates must receive vote share $\frac{A}{3}$. There are three ways the A candidates could tie: (I) all have the same policy; (II) exactly two candidates share the same policy; (III) all have different policies. Case I cannot form the basis for an equilibrium, because there must exist a potential A entrant who can win a vote share that is at least arbitrarily close to $\frac{A}{2}$ by continuity of F, which would give benefit $g(A) > c$ and therefore an incentive to enter. For both cases II and III, the largest vote share for the new A vote-winner after one candidate exits is $\frac{2A}{3}$ (for case II, when one of the coincident A candidates exits; for case III, when either of the A candidates who are at the "extremes" exits). If the vote share for the top A candidate exceeds B, then the A candidate wins; as such, there can be no identity incentive for exit only when $\frac{2A}{3} < B$, or $A < \frac{3}{5}$. Now consider a potential B entrant at the ideal point of the B incumbent candidate. Tying for first in an election is always worthwhile ($\frac{\gamma}{2} > c$), so such entry entrant can only be deterred if it would cause the now-tied B group candidates to place below the A candidates, i.e. if $\frac{B}{2} < \frac{A}{3}$ or $A > \frac{3}{5}$. As this is incompatible with the condition above, no equilibrium corresponds to cases II or III, and therefore to (5). This leaves (2), which is not feasible for all A: specifically, two A candidates can tie for the win only if $A \in (\frac{2}{3}, 1)$, whereas three A candidates can tie for the win only if $A \in (\frac{3}{4}, 1)$ because the B candidate must win vote share $1 - A$. The B candidate will wish to remain in the race for identity reasons (or simply because $g(B) > c$), and as B citizens have no identity or policy incentive to enter, they can be deterred from entry, for example if the B incumbent is at the median B voter and if $c > \frac{g(B)}{2}$. There is no identity incentive for A entry, and Proposition 3 of Osborne and Slivinski is sufficient to demonstrate that further A citizens can be deterred from entering for policy or winning reasons for either the two-way or three-way tie cases, since entrants in our models not motivated by identity must meet their conditions (as well as further constraints that are not necessary to consider). Finally, there is no identity incentive for A exit, and Proposition 3 of Osborne and Slivinski demonstrates that the further necessary and sufficient conditions can also be met. So $(3, 1)$ is possible for any $A \in (\frac{2}{3}, 1)$ and the possible equilibrium vote shares are as described. ■

Proof of Proposition 7. Same logic as in the corresponding plurality case. ■

Proof of Proposition 8. For $n > 1$, the proof is almost identical to that in the corresponding plurality case (Proposition 2). For $n = 1$, consider a potential group A entrant who shares the incumbent A candidate's ideal point. If she enters, she wins vote share $\frac{A}{2}$ in the first round. There are then three cases for the first round depending on A: (1) the A candidates tie for first; (2) the A candidates tie for second; and (3) all candidates tie for first. In (1), the two A candidates both advance to a runoff, which is also tied; each wins with probability $\frac{1}{2}$. In (2), each of the A candidates advances to (and then certainly wins) a runoff against B with probability $\frac{1}{2}$. In (3), with probability $\frac{1}{3}$, the two A candidates both advance to the runoff, which each wins with equal probability; in addition, each of the A candidates advances to (and then certainly wins) a runoff against B with probability $\frac{1}{3}$. In all three cases, the entrant wins with probability $\frac{1}{2}$. Because A candidate(s) always win(s) the election and maximum possible vote share regardless of entry, there are no identity costs or benefits to entry; and because here the A candidates share the same policy position, there are no policy costs or benefits to entry. The entry condition is then just $\frac{\gamma}{2} - c > 0$, which is true by assumption. So $(1, 1)$ is not possible. ■

Proof of Proposition 9. Take $A \in (\frac{1}{2}, \frac{2}{3})$. For $(2, 1)$ equilibrium existence the (i) A and (ii) B candidates must wish to stay in, and no other (iii) A or (iv) B candidates must wish to enter. (i) and (iii): The B candidate wins vote share $(1 - A) \in (\frac{1}{3}, \frac{1}{2})$ in the first round, earning no worse than second place, so the B candidate always makes it to the runoff against one ultimately victorious A opponent. Thus, being the top first-round A candidate is tantamount to election, and the strategic problem facing A candidates in the first round of the runoff system in a divided society is exactly the same as the one they would face in a *plurality* system in which the A group comprised the entire electorate. As such, Proposition 2 of Osborne and Slivinski, along with the observation that there are no identity reasons for A exit or entry, demonstrate that (i) and (iii) can can both be satisfied. (ii) The B candidate will clearly not wish to exit because $g(B) > c$ as well as for identity reasons. (iv) Group B entrants could be motivated either by group leadership concerns (which can be deterred by a B incumbent at the median B voter if $c > \frac{g(B)}{2}$) or by identity concerns. A solo B candidate in a runoff always loses; identity-motivated entry can occur here if and only if it leads *both* the B candidates to at least tie the top A candidate in the first round. The most efficient (and always feasible) allocation of B votes is to divide them equally between the B candidates, so deterrence of this case is necessary and sufficient for condition (iv). Since A candidates must tie in $(2, 1)$, the deterrence condition is $\frac{B}{2} < \frac{A}{2}$, which holds since $A > B$.

Now take $A \in (\frac{2}{3}, 1)$. Clearly the B candidate cannot tie or beat both of the A candidates. And, any A candidate trailing B does not make the runoff, so would wish to drop out to save entry costs. So either (1) both the A's beat the B in the first round or (2) one of the A's beats the B while the other ties. Two A candidates in a runoff must tie in the runoff, or the trailing candidate would drop out; so the A's either have the same policy or are symmetrically arranged around the overall median voter. An A always wins the election. For $(2, 1)$ equilibrium existence the (i) A and (ii) B candidates must wish to stay in, and no other (iii) A or (iv) B candidates must wish to enter. (i) There are clearly no identity reasons for A exit. For (1), $\frac{\gamma}{2} > c$ implies that there will be no incentive for exit; for (2), this will still be true so long as $\frac{\gamma}{4} > c$. (ii) The incumbent B candidate will clearly not wish to exit because $g(B) > c$ and for identity reasons. (iv) If the B incumbent is at the median B voter, all potential B entrants can be deterred so long as $c > \frac{g(B)}{2}$, as there are no identity motivations for entry in a situation where the sole B candidate was no better than tied for second to begin with. (iii) Consider (1). Suppose that the A incumbents have positions symmetric about the overall median voter. There are no identity incentives for entry since an A candidate always wins the election. For entry at a position that is either to the left or to the right of both A incumbents, there can be no policy incentive since entrants drain votes only from the incumbent whose policy the entrant prefers. An A incumbent would still make the runoff for sure, and would beat any such entrant who also made the runoff because of distance from the median voter. By familiar logic, it is also possible to deter entry between the two A incumbents; if the incumbents are sufficiently close together, such entrants would fail to make the runoff or change the composition of the candidates who do make the runoff. The remaining possibility is of a potential entrant at the policy of one of the incumbent candidates. An entrant at the policy of candidate A_j will receive vote share $\frac{A_j}{2}$; clearly the entry incentive will be at least as great at x_1 as at x_2 if $A_1 \geq A_2$ (which we assume without loss of generality). If $A_1 < 2A_2$, such an entrant would finish no better than a two-way tie for second in the first round: this best case scenario leads to a runoff place with probability $\frac{1}{2}$, and conditional on that a victory with probability $\frac{1}{2}$, for a best-case victory probability of $\frac{1}{4}$, and no impact on the probability distribution of policy outcomes, so that there will be an incentive to enter if and only if $\frac{\gamma}{4} > c$. As such, in this case, entry can be deterred if $\frac{\gamma}{2} > c > \frac{\gamma}{4}$. (If the entrant does worse than a two-way tie for second place, the expected winning and policy benefits of entry will both be at least weakly worse, so deterrence will be possibly for a weakly wider range of conditions.) If $A_1 = 2A_2$, then the entrant would be tied for first place among the A candidates, with an expected winning benefit $\frac{\gamma}{3}$ and an improved distribution of policy outcomes for the entrant. The deterrence condition here is $\frac{\gamma}{2} > c > \frac{\gamma}{3} + \frac{\delta}{6}$ which is clearly possible if the policy separation δ between the A incumbents is not too large. Finally, if $A_1 > 2A_2$, then the entrant would be tied for first, and win the runoff with probability $\frac{1}{2}$, so that there would always be an incentive for entry since $\frac{\gamma}{2} > c$. So in this case, entry cannot be deterred at all. As such, for (1), A entrants can be deterred as long as $A_1 \leq 2A_2$. (Therefore to determine the vote shares that are possible in equilibrium, considering A incumbents with identical positions is not necessary since $A_1 = A_2$ is already included here.) Now consider (2), with first-round vote shares $A_1 > A_2 = B$. Entry at the extremes of A_1 and A_2 and at x_1 and x_2 involve the same considerations and thus deterrence conditions as above. Entry in between A_1 and A_2 is not the same because now an infinitesimal measure of support garnered between A_1 and A_2 could potentially change the vote share orderings. Now suppose that all A_2's support comes from her "extreme" flank away from A_1, that A_1 gets from her "extreme" flank support less than A_2, that A_1's "centrist" support is at least three times closer to x_1 than to x_2, and that x_1 and x_2 are sufficiently close. Then there is no incentive for entry in between the incumbents (no chance to win since policies sufficiently close; and entrants cannot achieve policy improvements since the relevant voters are out of reach). So the conditions are the same in (2) as in (1). Thus, (i)-(iv) can be simultaneously satisfied, so $(2, 1)$ is possible under the conditions described. ∎

Proof of Proposition 10. An A candidate must win the election for sure; otherwise either A candidate would wish to drop out for identity payoff reasons. As such, the B candidates must tie each other; otherwise, trailing B candidates would wish to drop out, because group leadership payoffs provide the only incentive for entry. Also, an A candidate with no chance of winning would drop out, so both A candidates must make the runoff with some probability. And at least one A must be in the runoff every time as an A candidate must win for sure: so either (1) both A's beat the B's, or (2) one A beats the B's while the other A ties the B's. If both A candidates advance to the runoff, they must tie in the runoff (or one would wish to exit). For $(2,2)$ equilibrium existence the (i) A and (ii) B candidates must wish to stay in, and no other (iii) A or (iv) B candidates must wish to enter. (iv) There can be no identity motivation for B entry as all A candidates beat or tie all B candidates in the first round. Since B entrants cannot affect A policy choices, entry incentives are limited to B group leadership. The only way for tied B candidates to deter such entry is with candidates symmetrically spaced about the median. (ii) Identity motivations for exit can exist only if a B candidate's exit creates positive probability that two B candidates will simultaneously qualify for the runoff (if only one B candidate is in the runoff, she would lose for certain). For $(2,2)$ this cannot exist because exit leaves only one B candidate. As such, B candidates will not exit so long as $\frac{g(B)}{2} > c$. (i) There can be no identity incentive for A exit. If the two A candidates both defeat the B's outright, $\frac{\gamma}{2} > c$ implies there will be no incentive for exit; if one of the A candidates ties the B's, it is necessary and sufficient that $\frac{\gamma}{6} > c$, because this candidate makes the runoff with probability $\frac{1}{3}$ and conditional on that wins half the time. So it is possible for both A candidates to wish to stay in for either of cases (1) and (2). (iii) The argument in part (iii) of the proof of Proposition 9 for $A \in (\frac{2}{3}, 1)$ holds here, replacing B in that proof with $\frac{B}{2}$ here. Note that (i)-(iv) can be simultaneously satisfied, so $(2,2)$ is possible under the conditions described. ■

Proof of Proposition 11. For $A \in (\frac{1}{2}, \frac{2}{3})$, the argument in Proposition 9 for $(2,1)$, $A \in (\frac{1}{2}, \frac{2}{3})$, holds here except that the relevant reference in Osborne and Slivinski ("OS") is Proposition 3, and the deterrence condition for B entry is instead $\frac{B}{2} < \frac{A}{3}$, or $A > \frac{3}{5}$, for the three-way tie specified in OS Proposition 3, and $\frac{B}{2} < xA$, or $A > \frac{1}{2x+1}$ for the two-way tie (two A candidates get xA, $x \in (\frac{1}{3}, \frac{1}{2})$, while the third trails with $(1-2x)A$). Note $\frac{1}{2x+1} \in (\frac{1}{2}, \frac{3}{5})$ so that for the two-way tie A can take on any value between $\frac{1}{2}$ and $\frac{2}{3}$, as long as $x \in (max(\frac{1}{3}, \frac{1}{2A} - \frac{1}{2}), \frac{1}{2})$.

Now take $A \in (\frac{2}{3}, 1)$. Taking A candidate vote shares $A_1 \geq A_2 \geq A_3$, there are 20 different relative orderings (including potential indifference) of these vote shares along with that of the B candidate. The six with A_1 and A_2 unambiguously as the top two cannot be in equilibrium; if the last-placed A candidate exited, it would not affect who made the runoff, and therefore not affect policy, nor does the trailing A get identity or winning gains from staying in. The six with A_1 and B unambiguously as the top two also cannot be in equilibrium. A_2 and A_3 do not get winning or identity benefits from running, since an A candidate ultimately wins regardless, so only policy reasons could keep them from exiting. Only a candidate in the middle of three dispersed candidates could have such an incentive; extreme or coincident candidates can only draw support away from their most favored alternative. But clearly A_2 and A_3 cannot both be the central of three dispersed candidates, so at least one must wish to exit.

We consider the eight remaining orderings in turn. In each instance, the incumbent B candidate will not wish to withdraw because of identity reasons (and $g(B) > c$), and potential B entrants can be deterred, since there is no identity motive for entry (both B candidates cannot make the runoff since for $A > \frac{2}{3}$, $\frac{B}{2} < \frac{A}{3}$), if the B incumbent is at the median B voter, so long as $\frac{g(B)}{2} < c$. As such we consider only A candidate incentives below.

Three remaining orderings involve: (1) three A candidates tie for first place ($A > \frac{3}{4}$); (2) all four candidates tie for first place ($A = \frac{3}{4}$); and (3) three A candidates tie for second place ($A < \frac{3}{4}$). In their Proposition 8, OS describe alternative policy configurations leading to this vote share; to demonstrate existence here, it is sufficient to focus on a runoff equilibrium in which all three A candidates have different positions but win equal vote shares, and in which the two extreme candidates are symmetric about the (overall) median voter. Consider A exit incentives, noting there is no identity incentive for exit since an A always has to win. For (1), the analysis is identical to OS, and demonstrates that the entrants don't exit for $\frac{\gamma}{6} > c$. For (3), the A candidates compete for only one runoff spot. Each gets it with probability $\frac{1}{3}$, and after getting it, beats B in the runoff. So there will be no exit incentive here so long as $\frac{\gamma}{3} > c$. For (2), each of the A candidates competes in three of six possible runoff pairings; the central (non-central) candidate(s) win all of them (win one, tie one, and lose one), with no exit incentive so long as $\frac{\gamma}{2} > c$ ($\frac{\gamma}{4} > c$). Now consider incentives of potential A entrants. There are no identity-related motives for entry. OS show in their setting that entrants whose objective is to finish first or second among the A candidates can be successfully deterred. This is sufficient to show entry deterrence is possible here for (1), (2), and (3).

Two further orderings are (4) $A_1 > A_2 = A_3 > B$ and (5) $A_1 > A_2 = A_3 = B$. Consider x_1 and x_2 symmetric about the overall median voter, with $x_1 < x_3 < x_2$. Further suppose that the distribution of A_1 voters has support $[y, x_1]$, $y < x_1$; the distribution of A_2 voters has support $[x_2, z]$, $z > x_2$; and all A_3 voters are contained within $(\frac{3x_3+x_1}{4}, \frac{3x_3+x_2}{4})$. Here A_1 is always in the runoff. When $A_1 > 2A_2$, there is clearly no means of deterring entry (as an entrant at x_1 can win with probability $\frac{1}{2}$ and ensure her ideal policy), so we restrict our attention to the complementary cases. In (4), A_1 faces either of the other A candidates; A_1 ties A_2 but loses to A_3 in runoff matchups, so that A_1 and A_2 (A_3) win with probability $\frac{1}{4}$ ($\frac{1}{2}$). In (5), A_1 faces any of the other three candidates, beating B, tying A_2, and losing to A_3, so A_1 (A_2) $\{A_3\}$ win with probabilities $\frac{1}{2}$ ($\frac{1}{6}$), and $\{\frac{1}{3}\}$. For (4,5), one can write conditions for non-exit for all three A candidates in terms of these probabilities and the probabilities of victory that would hold if the candidates individually dropped out (which are clearly determined by the preference distribution described); γ; c; and the policy distances between candidates, which can clearly be satisfied simultaneously when γ is large enough relative to c and potential policy costs of entry. (Note also that the conditions for B can also be simultaneously satisfied.) For A entry, for the given preference distribution, no entrant can win or obtain identity benefit, and entrants who are able to win positive vote shares take them from their most favored candidate and therefore obtain no policy benefit. This establishes that A entry can be deterred for general (4) and (5) if and only if $A_1 < 2A_2$.

The final three orderings are (6,7) $B \geq A_1 = A_2 > A_3$ and (8) $A_1 > A_2 = B > A_3$. For (8), A_3 must clearly prefer the policy of A_1; if A_3 instead preferred the policy of A_2, she could not harm but might help A_2's prospects by dropping out of the race, and so would not wish to pay the costs of entry. Similarly label as A_1 the candidate whose policy A_3 prefers in (6,7) (without loss of generality). Consider x_1 and x_2 symmetric about the overall median voter, with $x_1 < x_3 < \frac{x_1+x_2}{2} < x_2$. Note first that for (8), as above, entry cannot be deterred if $A_1 > 2A_2$, so we take $A_1 < 2A_2$ (automatically true for (6,7)). Consider then a preference distribution for which vote share less than A_2 lies left of x_1; vote share less than A_2 lies right of x_1 but left of $\frac{3x_1+x_3}{4}$; vote share $y < A_2 - A_3$ lies right of x_2 while $A_2 - y$ lies left of x_2 but right of $\frac{x_3+3x_2}{4}$; and vote share A_3 lies on $[\frac{x_1+x_2}{2}, z]$ for some $z < \frac{3x_3+x_2}{4}$. In (6,7), A_1 and A_2 each win with probability $\frac{1}{2}$ while in (8) A_1 (A_2) wins with probability $\frac{3}{4}$ ($\frac{1}{4}$). For (6,7,8), one can write conditions for non-exit for all three A candidates in terms of these probabilities and the probabilities of victory that would hold if the candidates individually dropped out (which are clearly determined by the preference distribution described); γ; c; and the policy distances between candidates, which can clearly be satisfied simultaneously when c is small enough relative to the policy distances (so that A_3 will wish to enter to influence policy) and when γ is large enough relative to c and potential policy costs (for A_1 and A_2) of entry. (Note also that the conditions for B can also be simultaneously satisfied.) For A entry, for the given preference distribution, no entrant can make the runoff or obtain identity benefit, and entrants who are able to win positive vote shares take them from their most favored candidate and therefore obtain no policy benefit. This establishes that A entry can be deterred for general (6), (7), and (8) if and only if $A_1 < 2A_2$. ■